PROPORTIONAL REASONING 3
TEACHER EDITION

PROPORTIONAL REASONING APPLICATIONS

DASHBOARD*

*Login to the Teacher Portal at http://mathandteaching.org/essentials-proportional-reasoning/ for the live, up-to-date, dashboard.

ESTIMATED TIME (10 – 20 days)

3.0 Opening Problem: Twinkie, The Dog (1 day)
3.1 Proportional Reasoning (3 – 4 days)
Tech: Constant of Proportionality (0 – 1 day)
3.2 Best Buy Problems (2 – 3 days)
Tech: The Running Game (0 – 1 day)
3.3 Scale Drawings (2 – 3 days)
Tech: Marcellus the Giant (0 – 1 day)
3.4 Review (2 – 4 days)
Assessment (0 – 1 day)

CLASSROOM POWERPOINTS

PR3.0 Twinkie, The Dog
PR3.1 Art Supplies
PR3.1 Properties of Proportions
PR3.2 Socks
PR3.3 A Bird House

MATERIALS

- Rulers
- Markers
- Compasses [3.3]
- Quarter-inch graph paper (or use R6) [3.3]
- Plain large blank paper (11 X 17) [3.3]
- Large paper (for Poster Problems) [3.4]
- Colored pencils
- Scissors
- Calculators

TECHNOLOGY

Getting Started with Desmos (general resources)
DESMOS Constant of Proportionality [before 3.2]
DESMOS The Running Game [after 3.2]
DESMOS Marcellus the Giant [after 3.3]

REPRODUCIBLES

R7 Quarter-Inch Graph Paper [3.3]
R8 Matching Activity: Nuts! (1 per pair or group) [3.4]
R9 Constant of Proportionality (1 per student) [Tech]
(Answer Key)
R10 Running Game (1 per student) [Tech]
(Answer Key)
R11a-b Marcellus the Giant (1 per student) [Tech]
(Answer Key)

STUDENT RESOURCES

PR3 Student Packet
PR3 Text File for Translation
PR Definitions, Explanations, and Examples

PREPARE AHEAD

3.3 A Bird House – make a sample to use for demonstration

3.4 Poster Problems: Geometric Patterns
See Teaching Tips for directions.

3.4 Matching Activity: Nuts! Ask some students to cut up squares ahead of time and paper clip them together.

TEACHER RESOURCES

PR3 Teacher Edition (Front) (Answer Key)
PR3 Math Background
PR3 Teaching Tips

VIDEO Teaching Tips: Poster Problems (general resources)
VIDEO Math Background: About Proportions [3.1]
VIDEO Teaching Tips: A Best Buy Problem [3.2]
VIDEO Teaching Tips: Scale Drawings [3.3]

ASSESSMENTS

PR3 Quiz A (doc) (Answer Key)
PR3 Quiz B (doc) (Answer Key)

PARENT RESOURCES

PR3 Parent Letter

EXTRA PROBLEMS AND PRACTICE

PR Problem Bank (doc) (Answer Key)
Skill Boosters

OPTIONS FOR A SUBSTITUTE

Practice pages 1-7 may be completed independently any time after instruction.

3.4 Matching Activity: Nuts [after 3.3]
3.4 Vocabulary Activity [after 3.3]

COMMON CORE STATE STANDARDS

STANDARDS FOR MATHEMATICAL CONTENT

6.RP.A	**Understand ratio concepts and use ratio reasoning to solve problems.**
6.RP.3a	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
6.RP.3b	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, ~~tape diagrams,~~ double number line diagrams, or equations: Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
7.RP.A	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**
7.RP.2a	Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
7.RP.2b	Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
7.RP.2c	Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn*.
7.RP.2d	Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.
7.G.A	**Draw, construct and describe geometrical figures and describe the relationships between them.**
7.G.1	Solve problems involving scale drawings of geometric figures, including computing actual lengths ~~and areas~~ from a scale drawing and reproducing a scale drawing at a different scale.
7.EE.B	**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**
7.EE.3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
8.F.B	**Use functions to model relationships between quantities.**
8.F.4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

STANDARDS FOR MATHEMATICAL PRACTICE

Here are a few examples of how the Standards for Mathematical Practice are applied in these lessons.

MP1 Make sense of problems and persevere in solving them. [3.1, 3.2, 3.3] Students begin the Art Supplies problem and Socks problem pondering "what do you know" and "what do you wonder." This helps them make sense of problems. In the Bird House problem, students must figure out how to make a pattern from a scale drawing with tools of their choice.

MP2 Reason abstractly and quantitatively. [3.1] In the Properties of Proportions problem, students use double number lines and equivalent fractions to represent problem situations using symbols. Then they manipulate those symbols in search of a solution.

MP3 Construct viable arguments and critique the reasoning of others. [3.0, 3.3] Students predict whether Twinkie will break the record using mathematical reasoning. Students consider and explain strategies for making the birdhouse pattern.

MP4 Model with mathematics. [3.1, 3.2] Students experience situations where a proportional model does apply (such as in some of the Best Buy Problems) and where it does not apply (such as in Twinkie the Dog and in some Best Buy Problems).

MP5 Use appropriate tools strategically. [3.4] Students choose appropriate tools to make scale drawings.

MP6 Attend to precision. [3.2 3.3, 3.4] Students discuss appropriate scaling of graphs. Students measure, draw, and cut scale models where inaccurate measurements may lead faulty scale factor computations. This provides an opportunity to discuss measurement accuracy.

MP7 Look for and make use of structure. [3.1] In Properties of Proportions, students make, identify, and explain appropriate ways to manipulate these special equations.

MP8 Look for and express regularity in repeated reasoning. [3.1] In Properties of Proportions, students observe why the cross multiplication property for proportions is a valid and sense-making shortcut for solving equations of the form $\frac{x}{b}=\frac{c}{d}$.

STRATEGIES FOR SPECIAL POPULATIONS

Strategies for English Learners

Building background (Emphasize key vocabulary.) Be explicit when discussing unit rates. For example, in the Twinkie activity, emphasize that the unit rate refers to the number of balloons popped per second.

Instructional Strategies (Make concepts clear with visuals. Use scaffolding techniques.) Use arrows, circles, and color on double number lines to show proportional relationships that lead to valid proportions. Use animated powerpoints (such as Art Supplies) to guide "think aloud" discussions about how to create tables and double number lines to solve problems.

Instructional Strategies (Make concepts clear with visuals. Use scaffolding techniques.) Encourage students to use a variety of representations, including tables, double number lines, equivalent fractions, equations, and graphs, as some of these provide non-linguistic forms of communicating their understanding of proportional reasoning.

Student Interaction (Encourage elaborate responses. Provide wait time.) On all power points for the blank paper lessons (e.g. Twinkie, Socks, Bird house), encourage students to communicate with partners and the class using italic questions for oral discussions and numbered questions for written responses.

Practice/Review (Assess student comprehension. Provide student feedback on their output.) Poster Problems include opportunities for students to actively engage in their learning and to explain math ideas to each other. Listen carefully and provide feedback. This process allows for the development of their listening, speaking, reading, and writing skills.

Strategies for Special Learners

Know your learner (Understand student strengths and attributes that interfere with student learning). Watch for and address common misconceptions that students may have about proportional reasoning such as additive vs. multiplicative thinking.

Create a positive classroom culture (Make learning meaningful. Maintain high standards for all students.) Beginning with Twinkie, and then throughout this packet, students will encounter real-life examples and complex tasks that apply proportional ideas. Encourage students to dig into these problems.

Increase communication and participation (Monitor to ensure all students are benefiting from interactions. Allow alternative methods to express mathematical ideas.) Working with partners, sharing responsibilities, and talking through strategies and ideas are important for all lessons throughout this packet.

Make connections (Use multiple representations of math ideas—pictures, numbers, symbols, or words.) Students learn to use unit rates, tables, double number lines, equivalent fractions, equations, and graphs to solve proportional reasoning problems. Students should be able to interpret and present information in all formats, but encouraging students to rely more heavily on representation(s) that make sense will boost confidence as they improve in areas of weakness.

TEACHING TIPS

*Login to the Teacher Portal at http://mathandteaching.org/essentials-proportional-reasoning/ for more Teaching Tips.

Directions for Poster Problems (3.4)

Directions for Poster Problems are located in Teaching Tips in PR1.

Avoiding Learned Helplessness

When given contexts and solving problems, students may want immediate reassurance from an expert that their approaches and solutions are correct. Try to refrain from providing answers too quickly. For students to become independent, confident thinkers, they must be given adequate time to work through a problem, and sufficient opportunities to resolve misconceptions and unclear concepts through discussion with peers. Teachers unwittingly contribute to "learned helplessness" when they provide too much feedback without allowing, and demanding, adequate effort from students to work through problems themselves.

Address Misconceptions

Three common student misconceptions that emerge as students study proportional relationships are:

- Additive instead of multiplicative thinking (e.g. The ratios 3:4 and 4:5 are the same because 4 is one more than 3 and 5 is one more than 4).

  To correct, emphasize multipliers in arrow diagrams and use double number lines to show equivalence. Or present an extreme example by asking students to compare the ratios 1:2 and 99:100. ***Suppose one friend makes 1 out of every 2 free throws. The other makes 99 out of every 100. Are they equally skilled at making free throws? Which one would you rather have on your team in a basketball game?***

- Incorrect concepts of fractions (e.g., $\frac{3}{4}$, which is read "three over four," is interpreted as two numbers).

  To correct, emphasize this is "three fourths," or three copies of one-fourth.)

- Lack of relational thinking (e.g., in a table, students look only at a pattern from row to row, rather than attending to how two variables vary together).

  To correct, emphasize the co-variational relationship in representations. Or, have students work with tables where the independent variable does not always change by a fixed increment from one row to the next. Instead of x = 1, 2, 3, 4…, use x = 1, 2, 3, 7, 50, 77.

Look for faulty thinking as students solve problems and address these issues directly.

TECHNOLOGY AT A GLANCE

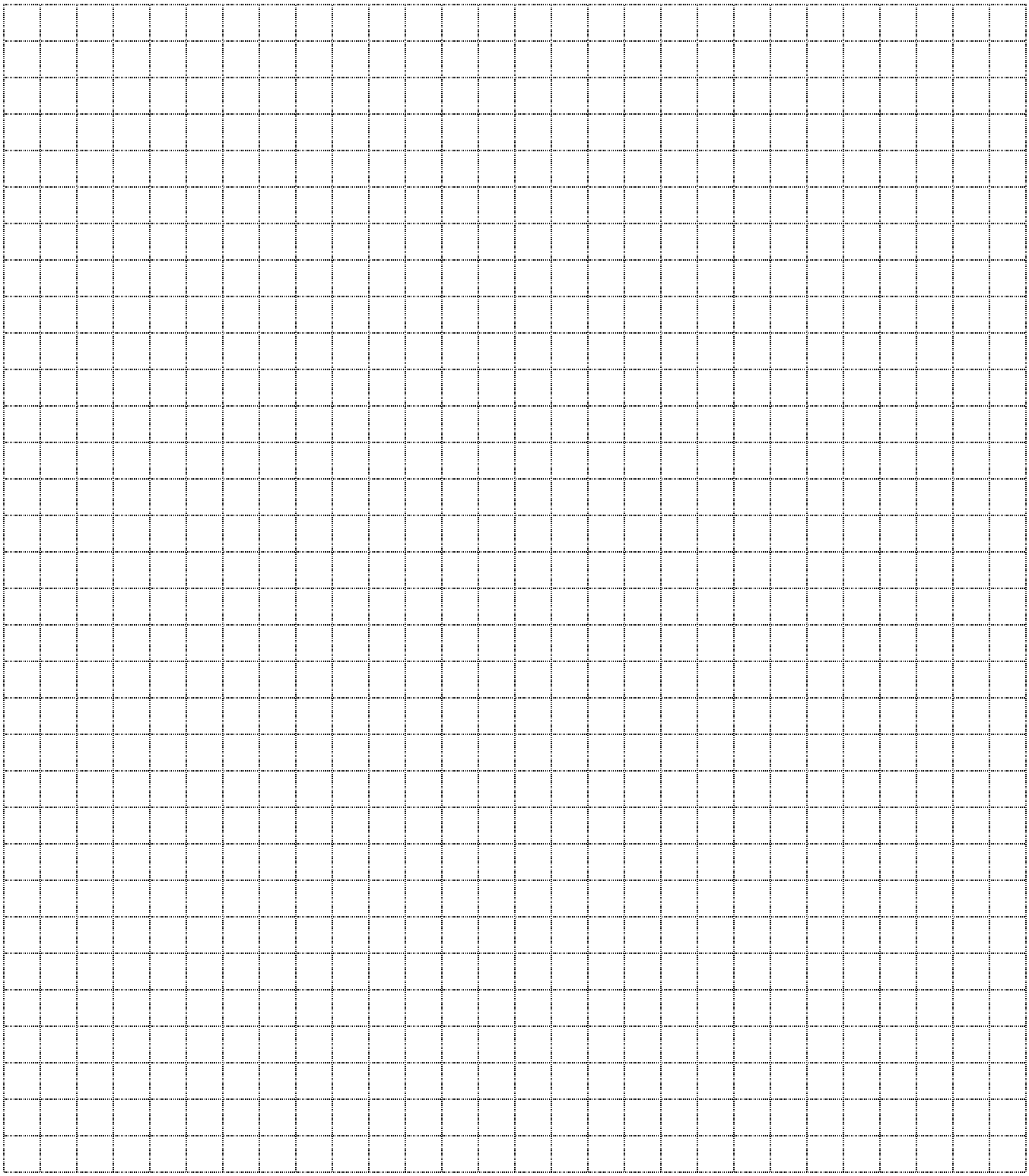
Technology activities in *MathLinks* aim to enhance the meaning of the content being studied and increase the engagement of students. *MathLinks* technology activities typically require software and hardware that is readily available in schools.

Log into the *MathLinks* teacher portal to download links to instructions and worksheets. (www.mathandteaching.org/login). All links are checked periodically, and we apologize in advance if third-party websites are taken down or links do not work.

Name of Activity	Technology	Description	Links
Constant of Proportionality	Desmos	Students create lines through the origin and find points on these lines. Then they describe the meaning of the points and the constant of proportionality in context of the problem. Use as an exploratory activity before lesson 3.2.	https://teacher.desmos.com/activitybuilder/custom/56097548686358ae072fff2d R9 Constant of Proportionality Worksheet Constant of Proportionality Worksheet Answer Key
The Running Game	Desmos	Students explore a potentially proportional relationship as they interpret pictures on stopwatches to predict running distances and times. Use after lesson 3.2.	https://teacher.desmos.com/activitybuilder/custom/563a5a1b3f80f2fd0b7c8074 R10 Running Game Worksheet Running Game Worksheet Answer Key
Marcellus the Giant	Desmos	Students explore scale factors and proportional relationships as they draw pictures of Dan and scale giants, and also analyze how graphs relate to these pictures. Use after lesson 3.3.	https://teacher.desmos.com/activitybuilder/custom/58093e7b37d6769f0b7fde92 R11a-b Marcellus the Giant Worksheet Marcellus the Giant Worksheet Answer Key

REPRODUCIBLES

R7 – QUARTER-INCH GRAPH PAPER



R8 – MATCHING ACTIVITY: NUTS!

Note: Each column below has four equivalent representations. Cut into 16 cards for students to match. (Note: The error in the "Mixed Nuts" table is intentional.)

TRAIL MIX 2 pounds for $12.00		**CHOCO NUTS** 4 pounds for $10.00		**MIXED NUTS** 3 pounds for $9.00		**FRUIT 'N NUTS** $\frac{1}{2}$ pound for $1.75	
# of lbs	**price in $**	**# of lbs**	**price in $**	**# of lbs**	**price in $**	**# of lbs**	**price in $**
2	12	2	5	2	6	2	7
4	24	4	10	4	16	4	14
0.5	3	0.5	1.25	0.5	1.5	0.5	1.75
1	6	1	2.5	1	3	1	3.5
Unit Rate $6 per pound		**Unit Rate** $2.50 per pound		**Unit Rate** $3.00 per pound		**Unit Rate** $3.50 per pound	
Equation Let x = # of lbs and y = price in $ $y = 6x$		**Equation** Let x = # of lbs and y = price in $ $y = 2.5x$		**Equation** Let x = # of lbs and y = price in $ $y = 3x$		**Equation** Let x = # of lbs and y = price in $ $y = 3.5x$	

desmos

R9 – THE CONSTANT OF PROPORTIONALITY

Go to student.desmos.com, get the class password from your teacher, and do the Desmos activity called Constant of Proportionality.

1. In the following table, what appears to be the constant of proportionality?

x	0	3	6	10	2.5	150
y	0	12	24	40	10	600

2. Given the following ordered pairs, what appears to be the constant of proportionality?

   (0, 0) (2, 5) (10, 25) (1, 2.5)

3. In as much detail as you can, describe the graph of a line with a constant of proportionality of $\frac{1}{2}$.

4. Write the numbers that might come next in this table, determine if there is a constant of proportionality, and explain your reasoning.

x	1	2	3	4	5	6	
y	1	4	9	16	25	36	

5. Go to lesson 3.2, Getting Started, and look at the information for Barter Jack's and Quigley's. Assume that at both stores you can buy any number of Healthy Crunch bars you like.

   1. Fill in tables to collect data on this product from these two stores.

   2. For each table, list the constant of proportionality (*k*), and describe whether this number is the same or different than the unit price (price per one bar).

Barter Jack's	
quantity	**price**
k =	

Quigley's	
quantity	**price**
k =	

desmos

R10 – THE RUNNING GAME

Go to student.desmos.com, get the class password from your teacher, and do the Desmos activity called The Running Game.

1. If Sam can run at a pace of 7:21 per mile on average, how long would it take him to run 4 miles at that pace?	2. Kim ran 3 miles in 30:04. What was her average pace per mile?

3. Amet ran 5 miles in 39 minutes. Each mile he ran at a different pace, but each mile was within 1 minute of the mile before it and the mile after it.

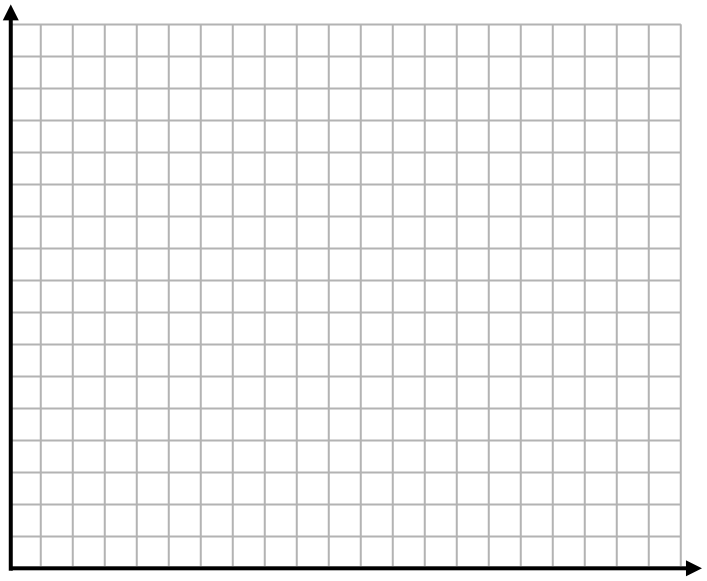
   a. Write reasonable times for each mile in the table.

Mile #	1	2	3	4	5
Time					

   b. Make a new table to record total time at each mile.

Distance in miles	1	2	3	4	
Time in minutes					39

   c. Graph the data in part b above. Label and scale your graph appropriately.



   d. Write a reasonable ordered pair to fit this graph:

      (6, _____)

      Explain what this ordered pair means in the context of the problem.

   e. Find a value that approximates this equation:

      Time in minutes = _____ • distance in miles

      Let t = time and d = distance, and rewrite the equation above: _____

desmos

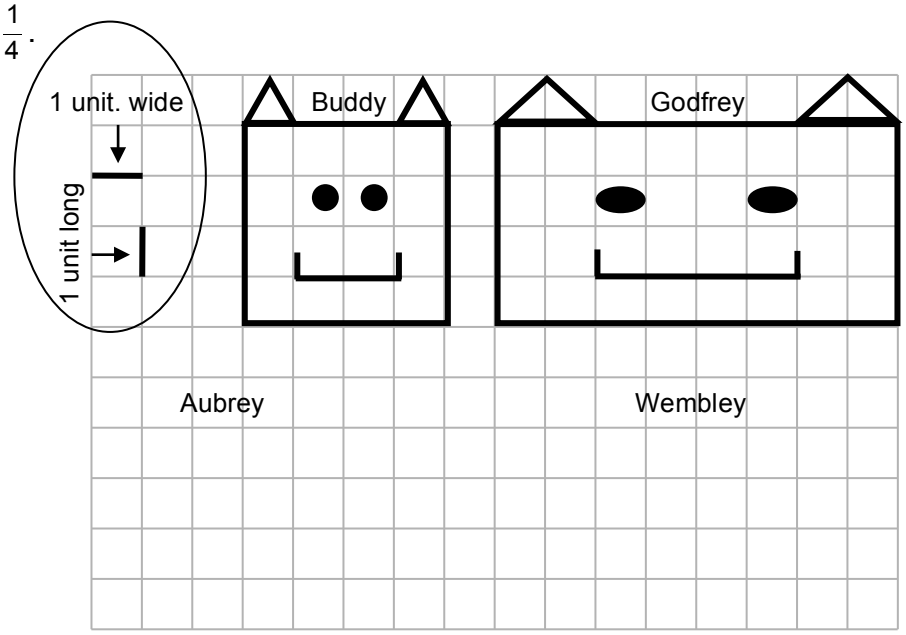
R11a – MARCELLUS THE GIANT

Go to student.desmos.com, get the class password from your teacher, and do the Desmos activity called Marcellus the Giant.

Go to lesson 3.3, Getting Started, and you'll find Buddy and his buddies. Aubrey and Wembley are other friends.

1. Draw Aubrey's face, which is proportional to Godfrey's face, multiplied by a scale factor of $\frac{1}{2}$.

2. Draw Wembley's face, which is also proportional to Godfrey's face, multiplied by a scale factor of $\frac{1}{4}$.



3. Fill in the table below. Pay close attention to how (width) and (length) are labeled.

	Face width	Ear width	Face length	Ear length	Mouth width
Buddy's dimensions					
Godfrey's dimensions					
Aubrey's dimensions					
Wembley's dimensions					

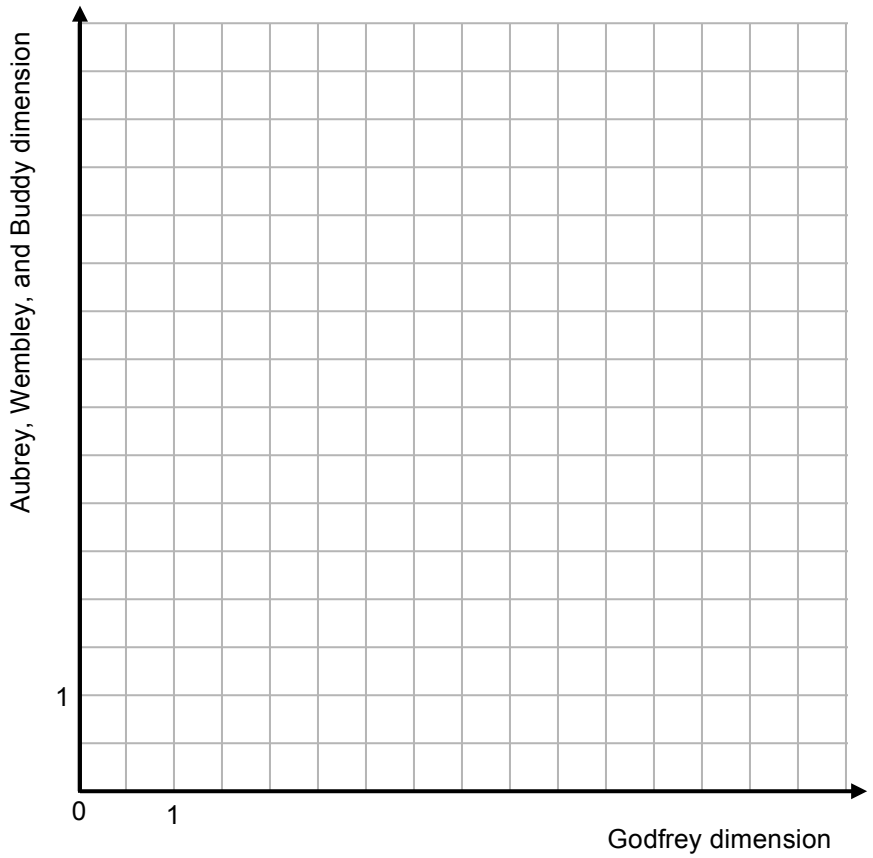
R11b – MARCELLUS THE GIANT

4. Draw the following graphs, comparing Godfrey's dimensions with the others. Use a different color for each.

   Graph 1 Ordered pairs in the form (Godfrey dimension, Aubrey dimension).

   Graph 2 Ordered pairs in the form (Godfrey dimension, Wembley dimension).

   Graph 3 Ordered pairs in the form (Godfrey dimension, Buddy dimension).



5. For which graphs do all five points lie on a straight line?

6. Which graphs are lines that go through the origin?

7. Which faces are proportional to Godfrey?